**Homework 8 Solutions due before you graduate**

**Problem 1.** Say a low-temperature solid’s energy and entropy were given by the following equations. Derive the formula for the pressure of the solid (when it’s in equilibrium).



We start with the first law, in differential form:



And then we consider the 2nd law, in differential form, taking account of the solid’s equipoise (i.e. dS­int. = 0):



Filling the dQ from the 2nd law into the dQ of the first law, we have:

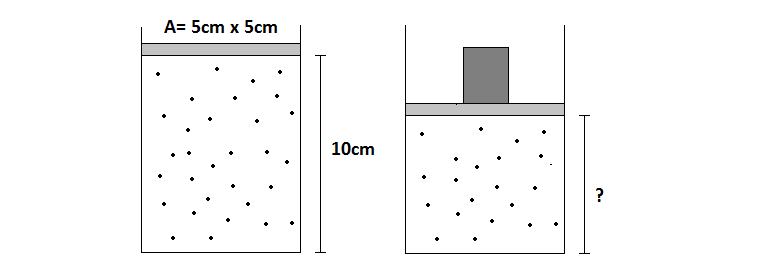


And now filling in our expressions for S and E:



So there you go.

**Problem 2.** Like everyone I know, you keep water vapor at 150°C in a well-insulated box with a movable piston at atmospheric pressure. But then, for no reason at all, you place a 5.1kg mass on top of it. (a) Assuming the gas remains well insulated so that no heat can escape or enter, what will be the new equilibrium height of the gas? (b) What will be the new temperature of the gas?



(a) The initial pressure was P1 = Patm = 101kPa. And the new pressure must be:



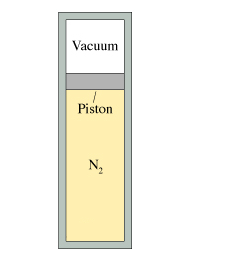
And the pressures are related via the isentropic equation of state



(b) To get the new temperature, we could use the other isentropic equation of state, the one that relates T and V together: T1f/2V1 = T2f/2V2. Then,



**Problem 3.** You’re the only one, to my knowledge, who keeps 0.09mol of N2 gas is in a container, supporting the weight of a 2kg piston. (a) If 15J of heat is poured into the gas, how high will the piston rise? (b) What will be its change in temperature? (c) What would be the efficiency of this process, i.e. η = W/Q? Though unnecessary, assume for simplicity the heat is poured in at such a rate to raise the weight at a constant speed.



(a) We have from the work-energy equation:



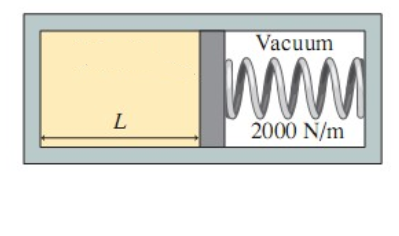
(b) Its change in temperature would be given by:



(c) The efficiency is η = W/Q = mgh/Q = (2)(9.8)(0.22)/15 = 29%.

**Problem 4.** A diatomic gas fills the left end of the cylinder. At 300K, the gas cylinder length is presently

L = 13cm, and the spring is compressed Δx = 5cm. How much heat must be added to the gas to expand the length to L = 16cm? You cannot assume a constant pressure from the gas now, because the force of the spring will increase as it compresses.



We can use the work energy equation:



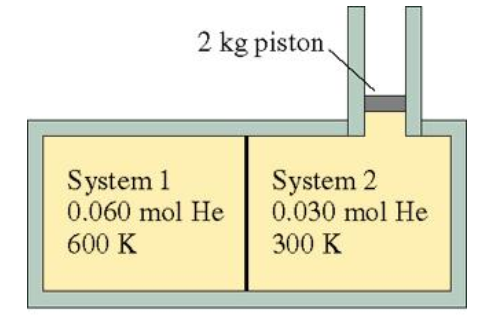
Now we need to get nRΔT. And we can use the ideal gas law, as usual. So Δ(pV) = nRΔT. Therefore,



Therefore,



**Problem 5.** When two objects come to equilibrium, the energy is usually wasted, but we could instead make a contraption to utilize some of that energy. Suppose two insulated compartments are separated by a thin wall. The left side contains 0.060mol of helium at an initial temperature of 600K and the right side contains 0.030mol of helium at an initial temperature of 300K. The volumes of the compartments are unknown, but the compartment on the right is attached to a vertical shaft. The 10cm diameter, 2.0kg piston can slide without friction up and down the shaft where the ambient external pressure is 1atm. Though unnecessary, we’ll also assume for simplicity that gas 2 will raise the piston at a constant rate.



(a) To what temperature will the gasses equilibrate? How does this compare to the temperature they’d equilibrate to if the piston couldn’t move?

We can use the 1st law, of course. I’ll apply it to the whole system,



Now we’ll use the fact that W2 (the work gas 2 does in raising the piston) is done isobarically, because the piston is raised at constant rate. That work would be W2 = p2ΔV2 = n2RΔT2 (from the ideal gas law). And so,



And this would be lower than the equilibrium temperature is the piston were fixed, since then energy wouldn’t have left the system as work, and would instead have been converted to thermal/internal energy.

(b) How high will the piston move in the process?

Here we return to p2ΔV2 = n2RΔT2 and solve for the change in volume, and relate to the height:



(c) How much heat was transferred to gas 2?

To get this we could apply the first law to gas 1 or gas 2 individually. It’ll be a little simpler if we apply it to gas 1. So we’d have:



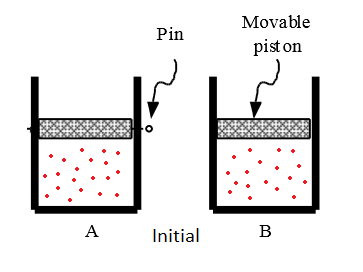
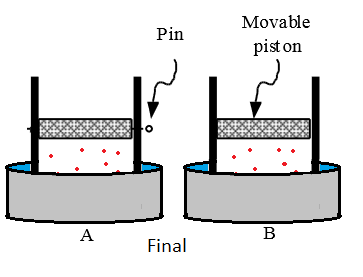
(d) How much work did gas 2 do?

Its work is just W = mgh = (2)(9.8)(0.053) = 1.04J.

(e) If we were to consider this an ‘engine’, what would be its efficiency?

This is η = W/Q = 1.04J/102J = 1% roughly.

**Problem 6.** Two identical gasses at the same temperature T > 0, same pressure p (supplied by the weight of the piston bearing down on them), and same number of particles, are placed in a bucket of ice. The gas on the left is not allowed to expand/contract due to the placement of a pin on the piston. But the gas on the right can move freely. After equilibrium between the gasses and ice is established, it is observed that there remains ice in the container, though less than before.

(a) How does the final temperature of A relate to the final temperature of B?

They are the same, since ice is left in each of them. So TA = TB = 0°C.

(b) How does the pressure of A relate to the pressure of B.

The pressure of B remains the same, since it is still supporting the piston. The pressure of A diminishes because the gas got colder, while its volume remained the same, which implies via p = NkT/V that p dropped. So pA < pB.

(c) How does the volume of A relate to the volume of B?

Well V = nRT/p. And since the temperatures of the two are equal, and pA < pB, we’ll have VA > VB.

(d) How does the amount of ice melted in A compare to the amount of ice melted in B?

In system A, energy is conserved and this results in some of the thermal energy of the gas going into the thermal energy of the ice and melting it, till the gas drops to temperature T = 0. In system B, the amount of ice melted is larger since the piston does work on the system. And since the change in energy of gas B is the same as that of gas A (because they drop to the same temperature, and that’s all the energy depends on for an ideal gas) the work done by the piston must have gone into the change in energy of the ice water. Therefore more melted.

Or in equations…we can apply the first law to system A, and to system B:

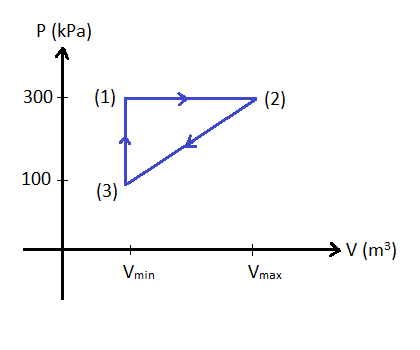


(Note ΔTA is negative so Δm is positive) And,



And since ΔTB = ΔTA, clearly Δm must be larger for system B than for system A.

**Problem 7.** Consider a steam engine, like the one we discussed in class, consisting of n = 3mol of water vapor. Let it operate between minimum/maximum temperatures T = 0°C, and T = 800°C, provided by cold and hot reservoir. And let it operate between minimum/maximum pressures p = 100kPa and p = 300kPa.



(a) Without doing any calculations, can you fill in the following table?

|  |  |  |  |
| --- | --- | --- | --- |
| **Process** | **1→2** | **2→3** | **3→1** |
| **Sign of W** | (+) | (-) | (0) |
| **Sign of Q** | (+) | (-) | (+) |

(a) What is Vmin?

The lowest temperature is the one that’s closest to the origin (point 3). So we have:



(b) What is Vmax?

The highest temperature is the one furthest from the origin (point 2). So,



(c) What is the temperature at point 1?

We can use same equation…



(d) What is the work done, and heat absorbed during process (1) → (2)?

Work is area under that line, so,



Heat can be calculated from 1st law:



(e) What is the work done and heat absorbed between process 2 → 3?

Work can be calculated as before, this time we’ll use the formula for the area under a trapezoid,



And to get heat, as usual 1st law:



(f) And what is the work done/heat absorbed between process 3 → 1?

This time, work is 0. And heat, comes from not the 2nd…not the 3rd…but the first law:



(g) What is total work done, heat absorbed, and heat exhausted?

Welp,



(h) What should be the change in energy of the engine during this complete cycle? And does this accord (sans rounding errors) with your calculations vis a vis Wtotal, Qh, Qc? This is a good check that you did the calculations correctly.

Total energy change of the engine should be zero since it returns to its same initial temperature. And our calculations accord with this since: Q – W = ΔE → (66.2 – 64.2) – 2.1 = 0 → -0.1 = 0, which is approximately true, neglecting tiny rounding error.

(i) What must be the change in entropy of the engine? And the reservoirs? Is the total positive (or zero at best) as the second law requires?

ΔSengine should be 0 since it returns to its initial state.

ΔShot reservoir is: ΔSh = -Qh/Th = -66.2kJ/1073K = -62J/K,

ΔScold reservoir is: ΔSc = Qc/Tc = 64.2kJ/273K = 235J/K

Total ΔS = 0J/K – 62J/K + 235J/K = 173J/K, which is positive.

(j) What is the efficiency of our engine? Evaluate via η = Wtotal/QH and η = (1-Tc/Th)(1+TcΔSint./Wtotal)-1.

This is:



And,



Of course they should match.

(k) How much heat would have to be input to raise a 500kg weight up through a vertical distance of 40m?

This would be:



**Problem 8**. That’s not so hot. Playing around with P-V cycle shapes, operating between the same two reservoirs, we construct an engine with 40% the maximum efficiency.

(a) What is its efficiency?

Max efficiency is given by



And so our new engine has an efficiency of:



(b) Our engine is being used instead to rotate a turbine, to generate electricity (via mechanism to be discussed perhaps in PHY 123). We need it to generate work at the rate of 5MW. What rate of heat input is required to power our engine? What is the rate of heat exhaustion to the environment (should use 1st law to connect work, heat input, and heat output)?

So then,



And from the 1st law:

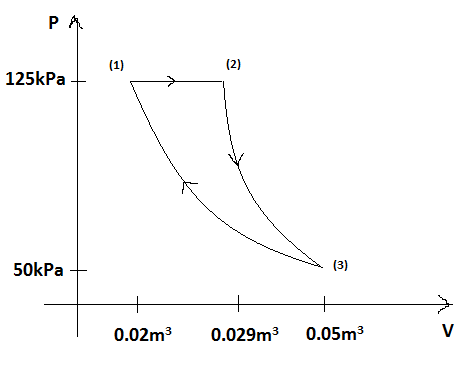


(c) The exhaust heat must be dumped somewhere. A nearby pond is a good choice! Suppose the pond has surface area A = 10 000m2, and depth h = 10m. The density of water is ρ = 1000 kg/m3, and its heat capacity is c = 4.18kJ/kg∙C. How many hours until the pond’s temperature increases by 1°C, assuming that it doesn’t lose any of the heat input by the engine.

The mass of water in the pond is m = ρV = 108kg. And so then, applying the first law to the pond,



**Problem 9**. Let an n = 0.7 mol monatomic gas be takeng through the following processes: (1) → (2) be isobaric, (2) → (3) be isentropic, and (3) → (1) be isothermal, at the pressures and volumes shown below. (a) What is the efficiency of this engine? (b) What would be the efficiency of a Carnot engine operating between the same two reservoirs?



(a) First we need to get the temperatures,



Then the work, heat done for each process is:



and,



and last,



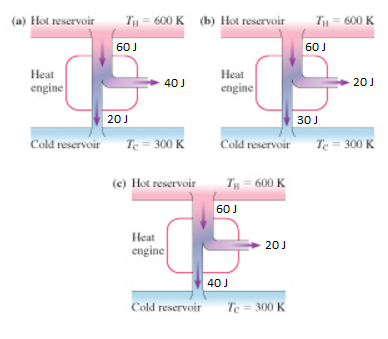
So the efficiency is:



(b) The Carnot efficiency is given by:



**Problem 10.** State which, if any, of the first and second laws of thermodynamics, the following engine cycles violate. Please justify ☺.



1. Since 60 = 40 + 20, the first law is satisfied, and since -60/600 + 20/300 = -0.033 < 0, the second law is violated.
2. Since 60 ≠ 20 + 30, the first law is violated. But since -60/600 +30/300 = 0 ≥ 0, the second law is barely satisfied.
3. Since 60 = 20 + 40, the first law I satisfied. And since -60/600 + 40/300 = 0.033 ≥ 0 the second law is satisfied.

**Problem 11.** You go out for a 6 mile jog. Ignoring internal work…

(a) How much work do you do?

Work is:



(a) Approximately how many kcalories do you burn?

This is,



(b) How much heat do you give off (assuming your temperature remains constant).

And this is,



**Problem 12**. Suppose you use a stair climber, and climb the equivalent of 850m. And further that your mass is about 70kg, your efficiency about 20%, and your heat capacity about 3.5 kJ/kg∙K. Ignore internal work…

(a) How many kcalories do you burn?

Calories burned is just



(b) If you wore a thick jacket to eliminate evaporative cooling and to minimize thermal radiation and conduction as means of cooling off how much would your core temperature have increased?

If you eliminate all heat transfer then we’ll have:



(c) If your core temperature increases about 3 degrees, and you lose heat through evaporation of sweat, how much water would you lose (in kg).

And then if we assume heat transfer via evaporation,



**Problem 13.** Suppose you’re outside in the cold (T = -20C). How thick a jacket will you need to stay warm (meaning your temperature doesn’t change) if its material has thermal conductivity k = 0.04 W/m∙K? You can take your temperature to be 35C. And you can suppose that the area of the jacket is A = 0.7m2. Just consider heat transfer via conduction.

So we have:



**Problem 14**. You can be modelled as a cylinder of height h = 1.7m, and radius R = 12cm, with skin temperature T = 35C. At what ambient temperature would you feel most comfortable? In other words, at what ambient temperature (in Farenheit) would you radiate heat at just the right pace to keep your temperature constant? Assume radiation is the only source of heat loss and takes place via all surfaces, and that ε = 1.

Yep.

